TRANSMISSION OF RADIATION ENERGY THROUGH ABSORBING AND DISPERSING MATERIALS IRRADIATED BY A PARALLEL BEAM AT SOME DEFINITE ANGLE OF INCIDENCE

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Laws governing the transmission of radiation energy are established on the basis of which it is possible to calculate the distribution of flux density and absorbed energy over the thickness of an absorbing and dispersing layer irradiated from both sides at some definite angle of incidence.

In [9] we have established laws which govern the attenuation of a diffuse radiation flux in materials. At the same time, infrared heat treatment and drying of food products and other materials is effected by various modes of irradiation, namely by two-sided or one-sided irradiation with a diffuse flux or with a wide parallel beam. Typical examples of parallel irradiation are infrared lamps with a parabolic mirror in an open chamber, or solar infrared radiation for drying vegetables, fruit, cotton, peat, etc. Various structural components made of radiation dispersing material are also exposed to solar infrared beams impinging at a continuously varying angle. Therefore, it is important to know the laws which govern the attenuation of a radiation beam impinging on a material at an angle.

It does not appear feasible to apply here known solutions to the equation of energy transmission for a narrow radiation beam through turbid media [1, 2, 6-8, 10-14], because they represent a special case with, above all, no data available on the dispersion indicatrix $\chi_\lambda(\gamma)$ and on the angular pattern of the radiation flux within a layer.

The difficulties in obtaining these data arise, because dispersion of radiation by inhomogeneities in materials under study is a more complex phenomenon than ordinary dispersion of radiation by particles. In this case the dispersion centers may be not only colloidal particles and density fluctuations, but also pores and capillaries randomly distributed in a body.

The propagation of a narrow beam of radiation through peat, wood, paper, food products, and other materials is peculiar in that it rapidly and almost entirely diffuses (or disperses) within a very thin layer. Experimental studies [4, 9] have shown that the fraction of flux which passes straight through a 0.1 mm layer of most materials is less than 5%. On the other hand, the hemispherical transmittance of the same specimen may be as high as 40-50%. Such a rapid conversion of a parallel beam into diffuse radiation has to do with strong and multiple dispersion at various optical inhomogeneities in the material layer.

During parallel irradiation, in a certain zone adjacent to the irradiated surface of the material at depth x there appear two fluxes of similar intensities: a parallel flux q'_λ and a derivative diffuse flux q_λ. In our case the problem concerning the attenuation of energy fluxes q'_λ and q_λ is expeditiously solved by the differential-difference method (discrete fluxes), which has been developed by Schuster, Schwarzschild, Duntley, Ambartsunyan, et al.

We will consider the process of energy transmission by monochromatic radiation through a plane layer of isotropic and selectively attenuating medium, width l. Parallel radiation beams $E'_{\lambda,1}$ and $E'_{\lambda,2}$ impinge on a layer from both sides, respectively, both at angle $\theta = \arccos\mu$ (Fig. 1). On the diagram we

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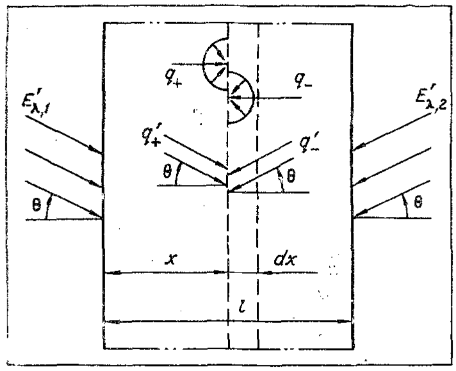

Fig. 1. Pertaining to the problem of a parallel radiation beam impinging at some angle and traveling through a plane layer of absorbing and dispersing material with an arbitrary dispersion indicatrix.

have blown up a layer element of thickness dx at depth x, its optical properties will be characterized by determinate empirical spectrum coefficients which do not require that the dispersion indicatrix and the spatial flux distribution inside the layer be known explicitly. Such characteristic coefficients will be the spectral absorptivity k_λ and dispersivity σ_λ with respect to parallel irradiation as well as the mean-angular absorptivity $\bar{k}_\lambda$, "forward" dispersivity f_λ, and "backward" dispersivity s_λ with respect to diffuse irradiation. Unlike the fundamental characteristics k_λ and σ_λ, the secondary mean characteristics, which can be determined experimentally, contain the overall information about the spatial distribution of radiation energy inside a layer as well as about the absorptive and the dispersive properties of the material. Information about the unknown third characteristic $\chi_\lambda(\gamma)$ and about the irradiation mode can be obtained through the special coefficients δ and m relating them to the fundamental optical absorption and dispersion characteristics as follows:

$$\bar{k}_\lambda = mk_\lambda, \tag{1}$$

$$s_\lambda = m\delta_s\sigma_\lambda, \quad f_\lambda = m\delta_f\sigma_\lambda. \tag{2}$$

The auxiliary coefficients δ and m, with the aid of which the mean characteristics of the material are plotted, can be found from known relations (see [3, 5, 7, 13]). The proportionality factor m, called the space distribution coefficient of incident radiation, is equal to the reciprocal of the mean cosine $\bar{\mu} = \overline{\cos\Theta}$ of the incidence angle Θ: $m = 1/\bar{\mu}$. When both beams impinge normally to the surface of the volume or layer element, then $m = 1$. For a hemispherical and perfectly diffuse incident flux $m = 2$.

Coefficients δ_s and δ_f are numerically equal to the forward fraction and the backward fraction, respectively, of flux dispersed in the volume or layer element with a dispersion indicatrix $\chi_\lambda(\gamma)$, when the incident radiation is contained within a solid angle $\omega' \le 2\pi$ with an angular intensity distribution $B_\lambda(\omega')$. We note that the sum of these two coefficients is always equal to unity: $\delta_s + \delta_f = 1$.

According to Fig. 1, layer dx is irradiated at an angle Θ by opposing parallel fluxes q'_+, q'_- and opposing diffuse fluxes q_+, q_- of the dispersed radiation.

Fraction $k_\lambda q'_\lambda$ of each parallel flux q'_+ and q'_- is absorbed by layer dx, while fraction $\sigma_\lambda q'_\lambda$ is dispersed in all directions according to the Buger–Lambert law. Moreover, the fraction $\delta_s\sigma_\lambda q'_\lambda = s'_\lambda q'_\lambda$ of total dispersed radiation is dispersed backward, while the fraction $\delta_f\sigma_\lambda q'_\lambda = f'_\lambda q'_\lambda$ is dispersed forward in the direction of the parallel flux impinging on layer dx.

Now, for the parallel fluxes q'_+ and q'_- we introduce the mean dispersivity in the forward direction $f'_+ = f'_- = f'_\lambda$ and in the backward direction $s'_+ = s'_- = s'_\lambda$, for the diffuse fluxes we introduce the mean absorptivity $\bar{k}_+ = \bar{k}_- = \bar{k}_\lambda$, the mean forward dispersivity $f_+ = f_- = f_\lambda$, and the mean backward dispersivity $s_+ = s_- = s_\lambda$.

Assuming that $m_+ = m_- = m \le 2$ and $\delta_+ = \delta_- = \delta \le 1$, with (1) and (2) taken into consideration, we can set up the following system of equations for q'_+, q'_-, q_+, and q_-:

$$\frac{dq'_+}{dx} = -\frac{1}{\mu}(k_\lambda + \sigma_\lambda)\,q'_+ = -\frac{\varepsilon_\lambda}{\mu}\,q'_+; \tag{3}$$

$$-\frac{dq'_-}{dx} = -\frac{1}{\mu}(k_\lambda + \sigma_\lambda)\,q'_- = -\frac{\varepsilon_\lambda}{\mu}\,q'_-; \tag{4}$$

$$\frac{dq_+}{dx} = -(\bar{k}_\lambda + s_\lambda)\,q_+ + s_\lambda q_- + \frac{f'_\lambda}{\mu}\,q'_+ + \frac{s'_\lambda}{\mu}\,q'_-; \tag{5}$$

$$-\frac{dq_-}{dx} = -(\bar{k}_\lambda + s_\lambda)\,q_- + s_\lambda q_+ + \frac{s'_\lambda}{\mu}\,q'_+ + \frac{f'_\lambda}{\mu}\,q_-, \tag{6}$$

with $\mu = \cos\Theta$.

Multiple dispersion in a plane layer irradiated with a parallel beam is taken into account by the system of two equations, (5) and (6) referred to dispersed fluxes, which supplement the Buger–Lambert law (3)-(4) of attenuation of parallel radiation. The boundary conditions will be

$$q'_{+}(x=0)=E'_{\lambda,1},\quad q_{+}(x=0)=0,$$
$$q'_{-}(x=l)=E'_{\lambda,2},\quad q_{-}(x=l)=0. \tag{7}$$

The solution to system (3)-(6) with boundary conditions (7) is

$$q'_{+}=E'_{\lambda,1}\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right), \tag{8}$$

$$q'_{-}=E'_{\lambda,2}\exp\left[-\frac{\varepsilon_\lambda}{\mu}(l-x)\right], \tag{9}$$

$$\begin{aligned}q_{+}=&\frac{E'_{\lambda,1}}{1-\Psi_\lambda^2}\Big\{C_2\left[\exp(-L_\lambda x)-\Psi_\lambda^2\exp(L_\lambda x)\right]+C_1\Psi_\lambda\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\\&\times\left[\exp(L_\lambda x)-\exp(-L_\lambda x)\right]\Big\}-E'_{\lambda,1}C_2\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right)\\&+\frac{E'_{\lambda,2}}{1-\Psi_\lambda^2}\Big\{C_2\Psi_\lambda\left[\exp(L_\lambda x)-\exp(-L_\lambda x)\right]+C_1\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\\&\times\left[\exp(-L_\lambda x)-\Psi_\lambda^2\exp(L_\lambda x)\right]\Big\}-E'_{\lambda,2}C_1\exp\left[-\frac{\varepsilon_\lambda}{\mu}(l-x)\right];\end{aligned} \tag{10}$$

$$\begin{aligned}q_{-}=&\frac{E'_{\lambda,2}}{1-\Psi_\lambda^2}\Big\{C_2\left[\exp\{-L_\lambda(l-x)\}-\Psi_\lambda^2\exp\{L_\lambda(l-x)\}\right]\\&+C_1\Psi_\lambda\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\left[\exp\{L_\lambda(l-x)\}-\exp\{-L_\lambda(l-x)\}\right]\Big\}\\&-E'_{\lambda,2}C_2\exp\left[-\frac{\varepsilon_\lambda}{\mu}(l-x)\right]+\frac{E'_{\lambda,1}}{1-\Psi_\lambda^2}\Big\{C_2\Psi_\lambda\left[\exp\{L_\lambda(l-x)\}-\exp\{-L_\lambda(l-x)\}\right]\\&+C_1\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\left[\exp\{-L_\lambda(l-x)\}-\Psi_\lambda^2\exp\{L_\lambda(l-x)\}\right]\Big\}-E'_{\lambda,1}C_1\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right),\end{aligned} \tag{11}$$

where

$$\Psi_\lambda=R_{\lambda\infty}\exp(-L_\lambda l); \tag{12}$$

and C_1, C_2 are parameters which describe the optical properties of the medium as well as the spatial energy distribution of fluxes q_+, q_- at depth x and which account for the mode of layer irradiation:

$$C_1=\frac{\mu s'_\lambda\bar{\varepsilon}_{\lambda,9}-s'_\lambda\varepsilon_\lambda+\mu f'_\lambda s_\lambda}{\varepsilon_\lambda^2-\mu^2L_\lambda^2}=\frac{(1-\delta_f)(m\mu-1)\Lambda}{1-\mu^2K^2}; \tag{13}$$

$$C_2=\frac{\mu f'_\lambda\bar{\varepsilon}_{\lambda,9}+f'_\lambda\varepsilon_\lambda+\mu s'_\lambda s_\lambda}{\varepsilon_\lambda^2-\mu^2L_\lambda^2}=\frac{(1+m\mu)\delta_f\Lambda+m\mu(1-2\delta_f)\Lambda^2}{1-\mu^2K^2}; \tag{14}$$

$$K=m\sqrt{(1-\Lambda)\left[1+\Lambda(1-2\delta_f)\right]}=\frac{L_\lambda}{\varepsilon_\lambda}. \tag{15}$$

Both L_λ and $R_{\lambda\infty}$ are related to the decay factor ε_λ and to the life expectancy of a quantum $A=\sigma_\lambda/\varepsilon_\lambda$ according to

$$L_\lambda=m\varepsilon_\lambda\sqrt{(1-\Lambda)\left[1+\Lambda(1-2\delta_f)\right]}=\sqrt{\bar{k}_\lambda(\bar{k}_\lambda+2s_\lambda)}, \tag{16}$$

$$R_{\lambda\infty}=\frac{1-\delta_f\Lambda-\sqrt{(1-\Lambda)\left[1+\Lambda(1-2\delta_f)\right]}}{(1-\delta_f)\Lambda}, \tag{17}$$

based on Eqs. (1)-(2).

With the aid of expressions (8)-(17) based on mean characteristics, without information about $\chi_\lambda(\gamma)$ and about the spatial distribution of radiation energy in the layer, one can determine the monochromatic diffuse radiation fluxes at depth x in a material layer of thickness l with an arbitrary dispersion indicatrix and irradiated from both sides at an angle Θ. These expressions will be more general than those derived in [13] for the special case of one-sided irradiation with the energy of perfectly diffuse fluxes q_+, q_- distributed uniformly in space (m = 2).

For diffuse irradiation of a layer, expressions (10) and (11) simplify into expressions obtained for this case earlier in [9].

The total density vector of monochromatic radiation flux has the scalar value

$$q'_\lambda(x)=(q'_+-q'_-)+(q_+-q_-). \tag{18}$$

The quantity of radiation energy per unit time absorbed by a volume element of thickness dx at depth x is determined from the equation representing the law of energy conservation

$$w'_\lambda(x)=k_\lambda(q'_++q'_-)+\bar{k}_\lambda(q_++q_-)=k_\lambda E'_{\lambda,0,\mathrm{B}}+\bar{k}_\lambda E_{\lambda,0}. \tag{19}$$

Expressions (18) and (19) for w'_λ and q'_λ become unwieldy for the general case.

When both impinging fluxes are equal, $E'_{\lambda,1}=E'_{\lambda,2}=E'$, then the formulas for q'_λ and w'_λ become much simpler:

$$q'_\lambda(x)=E'_\lambda\frac{1-R_{\lambda\infty}}{1+\Psi_\lambda}\{\exp(-L_\lambda x)-\exp[-L_\lambda(l-x)]\}\times\left[C_2+C_1\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\right], \tag{20}$$

$$w'_\lambda(x)=\bar{k}_\lambda E'_\lambda\frac{1+R_{\lambda\infty}}{1+\Psi_\lambda}\{\exp(-L_\lambda x)+\exp[-L_\lambda(l-x)]\}\times\left[C_2+C_1\exp\left(-\frac{\varepsilon_\lambda}{\mu}l\right)\right]-\bar{k}_\lambda E'_\lambda(C_1+C_2)$$
$$\times\left\{\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right)+\exp\left[-\frac{\varepsilon_\lambda}{\mu}(l-x)\right]\right\}+k_\lambda E'_\lambda\left\{\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right)+\exp\left[-\frac{\varepsilon_\lambda}{\mu}(l-x)\right]\right\}. \tag{21}$$

The quantity of energy absorbed in a layer of an infinite optical thickness is

$$w'_\lambda(x)=\bar{k}_\lambda E'_\lambda\left[(1+R_{\lambda\infty})C_2\exp(-L_\lambda x)+(C_1+C_2)\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right)\right]$$
$$+k_\lambda E'_\lambda\exp\left(-\frac{\varepsilon_\lambda}{\mu}x\right)=\bar{k}_\lambda E'_\lambda E^*_{\lambda 0}+k_\lambda E'_\lambda E^{*\prime}_{\lambda,0,\mathrm{B}}. \tag{22}$$

For irradiation at an angle Θ, the thermoradiation characteristics $R_\lambda(\Theta;2\pi)=R'_\lambda$ and $T_\lambda(\Theta;2\pi)=T'_\lambda$ are found from expressions (8)-(11) in the case of diffuse fluxes q_+, q_- and parallel fluxes q'_+, q'_-, respectively, with one-sided irradiation ($E'_{\lambda,2}=0$).

With the fraction of forward flux q'_+, according to Buger's law, $T'_\mathrm{B}=\exp(-\varepsilon_\lambda l/\mu)$ and with the known expressions for T_λ and R_λ (see [3, 5, 7, 9]), one can write down for a layer of material with an arbitrary dispersion indicatrix the following expressions

$$R'_\lambda=C_2R_\lambda-C_1(1-T'_\mathrm{B}T_\lambda), \tag{23}$$

$$T'_\lambda=C_2(T_\lambda-T'_\mathrm{B})+T'_\mathrm{B}(1+C_1R_\lambda, \tag{24}$$

$$A'_\lambda=1-\{C_2(R_\lambda+T_\lambda-T'_\mathrm{B})-C_1[1-T'_\mathrm{B}(R_\lambda+T_\lambda)]+T'_\mathrm{B}\}, \tag{25}$$

which relate its thermoradiation characteristics R'_λ, T'_λ, A'_λ referred to irradiation by a parallel beam at angle Θ and R_λ, T_λ, A_λ referred to irradiation by a diffuse flux.

The spectral reflectivity of a layer with an infinite optical thickness is

$$R'_{\lambda\infty}=C_2R_{\lambda\infty}-C_1. \tag{26}$$

Formulas of the (23)-(24) type (for normal incidence $\mu=1$) were first derived by Duntley [17], who experimentally demonstrated their applicability to paper. Analogous formulas have been derived in [13, 19] for the case of perfect diffusion (m = 2) of dispersed fluxes q_+, q_- inside the layer.

For the extreme case of an only absorbing medium ($\sigma_\lambda=0$, $\Lambda=0$), $C_1=0$, $C_2=0$, and from (24) for $\mu=1$ we have the known expression representing Buger's law:

$$T'_\lambda(\sigma_\lambda=0)=\exp(-k_\lambda l). \tag{27}$$

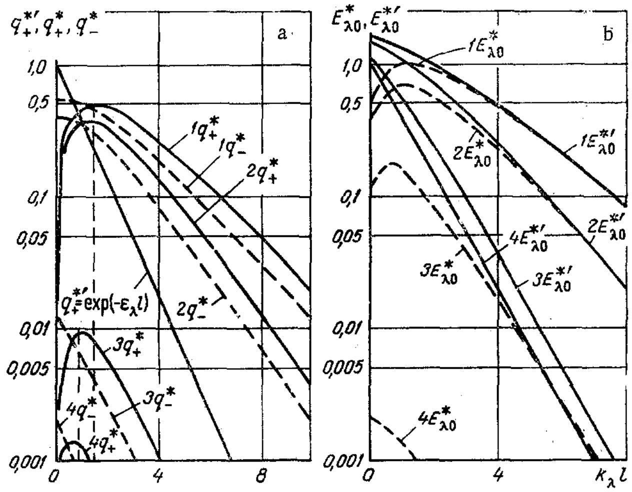

Fig. 2. a) Dimensionless parallel flux $q_+^{*\prime}$ and dispersed fluxes q_+^*, q_-^*; b) spatial irradiance due to dispersed fluxes $E_{\lambda,0}^* = q_+^* + q_-^*$ and total spatial irradiance $E_{\lambda,0}^* = q_+^* + q_-^* + q_+^{*\prime}$, as functions of the optical thickness $k_\lambda l$, at various values of the dispersion factor Λ: 1) 0.95; 2) 0.9; 3) 0.5; 4) 0.1.

All these formulas for q_+, q_-, $q_\lambda^\prime$, $w_\lambda^\prime$, and the expressions relating the mean characteristics $\bar{k}_\lambda$, s_λ, f_λ, L_λ with the basic characteristics k_λ, σ_λ, $\chi_\lambda(\gamma)$ make it possible to analyze the radiation field inside a plane layer of material irradiated by a wide parallel beam.

In Fig. 2 are shown the dimensionless magnitudes of fluxes $q_+^* = q_+/E_\lambda^\prime$, $q_-^* = q_-/E_\lambda^\prime$ generated by the dispersion of opposing diffuse fluxes and the spatial irradiances $E_{\lambda 0}^{*\prime}$ and $E_{\lambda 0}^*$, as functions of the dispersion characteristic Λ of the medium and of its optical thickness $k_\lambda l$ in a semiinfinite layer under normal irradiation ($\mu = 1$), for the special case of a symmetric dispersion indicatrix ($\delta_f = 0.5$). All curves here have a complex shape, deviating from an exponential curve at small optical thicknesses ($k_\lambda l < 3$) and approaching it at large optical thicknesses. As $k_\lambda l$ increases, at any value of Λ, the density of dispersed radiation q_+^* and the optical irradiance $E_{\lambda 0}^*$ both increase in the boundary zone, approaching their maxima at certain definite values of $k_\lambda l_m$, and then decrease again in some complex manner.

The maxima of functions $q_+^*(x)$ and $E_{\lambda 0}^*(x)$ can be located with the aid of relations (10) and (11) with (19) taken into account. In the case of a semiinfinite layer we have for function $E_{\lambda 0}^*(x)$

$$k_\lambda l_m (E_{\lambda 0}^*) = \frac{1}{\mu K - 1} \ln \left[\mu K \frac{(1+R_{\lambda\infty}) C_2}{1+C_1} \right]. \tag{28}$$

It is evident from the diagram that, as Λ increases, the maxima of q_+^* and $E_{\lambda 0}^*$ shift toward large optical depths. The intensity of dispersed radiation increases here continuously, moreover, owing to the dispersion of parallel radiation, because the fluxes are spatially not discretized within this range. The disperse irradiance is mainly due to a single dispersion of a parallel beam. There follows a range where q_+^* and $E_{\lambda 0}^*$ decrease quasiexponentially with increasing depth, where the dispersed radiation is essentially propagated outside the geometrical zone of a parallel beam. Very deep inside the layer ($k_\lambda l \gg k_\lambda l_m$) there occurs multiple dispersion, which ensures additional pumping of energy into the flux in violation of Buger's law of attenuation [7].

In the case of highly dispersive media ($\Lambda > 0.9$, curves 1) the spatial irradiance $E_{\lambda,0}$ and the flux density q_+ at depths $k_\lambda l > k_\lambda l_m$ are determined mainly by the dispersed radiation flux. The parallel flux $q_+^\prime$ is smaller than the diffuse flux q_+ and thus $T_B^\prime \ll T_\lambda$. Expressions (23) and (24) simplify then considerably:

$$R_\lambda^\prime = C_2 R_\lambda - C_1 = C_2 \frac{R_{\lambda\infty}[1-\exp(-2L_\lambda l)]}{1-R_{\lambda\infty}^2 \exp(-2L_\lambda l)} - C_1, \tag{29}$$

$$T_\lambda^\prime = C_2 T_\lambda = C_2 \frac{(1-R_{\lambda\infty}^2)\exp(-L_\lambda l)}{1-R_{\lambda\infty}^2 \exp(-2L_\lambda l)}. \tag{30}$$

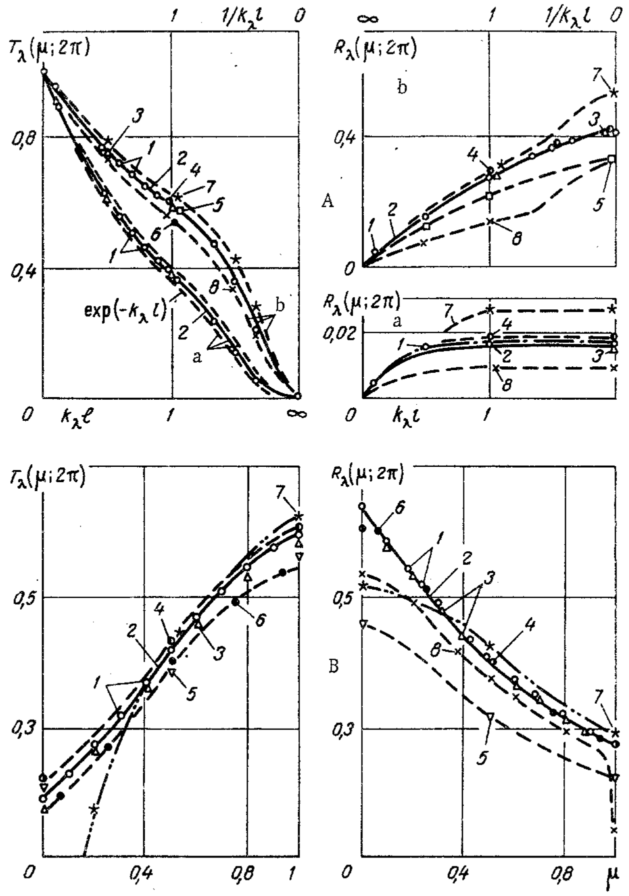

Fig. 3. Hemispherical reflectance $R_\lambda(\mu; 2\pi)$ and hemispherical transmittance $T_\lambda(\mu; 2\pi)$: A) function of the optical thickness $k_\lambda l$ at fixed values of the dispersion factor $\Lambda = 0.1$ (a) and 0.9 for (b), for $\mu = 1$; B) function of the cosine of the incidence angle μ ($\mu = \cos\Theta$), at $k_\lambda l = 1$ and $\Lambda = 0.9$, obtained by various methods: 1) according to formulas (23) and (24); 2) numerical method [20]; 3) numerical method (Hottel) [18]; 4) diffusion method [20]; 5) diffusion method (Richards) [21]; 6) method of spherical harmonics [8]; 7) two-fluxes approximation [16]; 8) six-fluxes approximation [16].

Formulas (29) and (30), first derived by Duntley [17] and later by Lathrop [19], make it possible, when $\mu = 1$, to relate the thermoradiation characteristics of the layer R'_λ, T'_λ to the optical characteristics of the medium k_λ, $\bar{k}_\lambda$, s_λ, L_λ and to determine the parameters C_1, C_2 on the basis of R'_λ, T'_λ and R_λ, T_λ measured under parallel and diffuse irradiation, respectively.

A simultaneous solution of (23) and (24) with respect to $L_\lambda l$ for the case where $\exp(-\varepsilon_\lambda l/\mu) \to 0$ yields the following expressions relating the optical characteristics of the medium to the thermoradiation characteristics of the layer R'_λ, T'_λ under parallel irradiation:

$$L_\lambda = \frac{1}{l} \ln\left[\frac{(R'_{\lambda\infty} - R'_\lambda)(R'_{\lambda\infty} - C_1)}{C_2 T'_\lambda}\right], \tag{31}$$

$$\bar{k}_\lambda = \left[\frac{C_2 - (R'_{\lambda\infty} - C_1)}{C_2 + (R'_{\lambda\infty} - C_1)}\right] L_\lambda, \tag{32}$$

$$s_\lambda = \left[\frac{2(R'_{\lambda\infty} - C_1)}{C_2 - (R'_{\lambda\infty} - C_1)^2}\right] L_\lambda. \tag{33}$$

TABLE 1. Values of $R_\lambda(\mu;\ 2\pi)$ and $T_\lambda(\mu;\ 2\pi)$ in the Case of a Symmetric Indicatrix $\chi_\lambda(\gamma)$, $k_\lambda l = 1$, and $\Lambda = 0.9$, for Various Values of μ

Method of determination	$\mu = 0$		$\mu = 0.5$		$\mu = 0.1$	
	R'_λ	T'_λ	R'_λ	T'_λ	R'_λ	T'_λ
According to formulas (23) and (24)	0.632	0.189	0.403	0.422	0.274	0.601
Numerical method [20]	0.635	0.182	0.393	0.415	0.267	0.593
Numerical method (Hottel approximation) [18]	0.636	0.173	0.400	0.418	0.282	0.585
Diffusion method [20]	0.600	0.220	0.398	0.431	0.273	0.602
Method of spherical harmonics (first approximation) [8]	0.63	0.17	0.39	0.400	0.275	0.540
Method of moments (second approximation) [20]	0.63	0.18	0.39	0.415	0.27	0.59

TABLE 2. Values of $R_{\lambda\infty}(\mu;\ 2\pi)$ in the Case of a Symmetric Dispersion Indicatrix $\chi_\lambda(\gamma)$, for Various Values of Λ and μ

Method of determination	$\Lambda = 0.1$			$\Lambda = 0.9$		
	$\mu = 0$	$\mu = 0.5$	$\mu = 0.1$	$\mu = 0$	$\mu = 0.5$	$\mu = 1.0$
According to formula (26)	0.051	0.026	0.017	0.683	0.518	0.435
Numerical method [20]	0.051	0.024	0.016	0.684	0.508	0.415
Numerical method (Ambartsumyan) [2, 7]				0.68	0.51	0.43
Diffusion method [20]	0.048	0.026	0.018	0.659	0.517	0.426
Method of spherical harmonics (third approximation) [20]	0.049	0.025	0.017	0.673	0.512	0.418
Method of moments (second approximation) [20]	0.051	0.024	0.016	0.671	0.489	0.403

TABLE 3. Values of $R_{\lambda\infty}(\mu;\ 2\pi)$ in the Case of a Forward Elongated Indicatrix $\chi_\lambda(\gamma) = 1 + \cos\gamma$ and Normal Incidence ($\mu = 1$), for Various Values of Λ

Method of determination	Λ					
	0.4	0.5	0.6	0.7	0.8	0.9
According to formula (26)	0.052	0.073	0.107	0.145	0.212	0.338
Numerical method [7]	0.04	0.06	0.09	0.13	0.20	0.33
Differential-difference method [13]	0.05	0.08	0.11	0.15	0.22	0.32
Method of moments [7, 10]	0.053	0.076	0.109	0.151	0.223	0.349

The decay factor ε_λ can be found from (23) and (24) but only in the case of small optical thicknesses, where $T'_B > 0$.

In order to evaluate the accuracy of the described procedure for analyzing the transmission of radiation energy through dispersing and absorbing materials under parallel irradiation, values of $R_\lambda(\mu;\ 2\pi)$, $T_\lambda(\mu;\ 2\pi)$, and $R_{\lambda\infty}(\mu;\ 2\pi)$ calculated by various methods are shown in Fig. 3A, B and in Tables 1-3. It is evident here that the values of reflectance and transmittance obtained by our method of discrete fluxes and by numerical methods of Hottel, Tien, Churchill, et al. [18, 20] for various incidence angles Θ, various dispersion characteristics of the medium Λ, and various optical thicknesses $k_\lambda l$ differ on the average by 1-2%. This indicates that the proposed method is sufficiently accurate when applied to capillary-porous colloidal materials.

Curves 1 (Fig. 3A) of $T_\lambda(\mu;\ 2\pi)$ as a function of $k_\lambda l$ at $\Lambda = 0.1$ and 0.9, calculated by formula (24), agree within 1-2% with curves 2, 3 obtained by numerical methods [18, 20] within the range of optical thicknesses $k_\lambda l$ from 0 to 3. For $k_\lambda l > 3$ curves 1 gradually depart toward small values of T_λ and approach curve 8 representing the six-fluxes Chu–Churchill approximation [16]. The reflectance as a function of $k_\lambda l$ agrees within 1-2% with the relations obtained by numerical methods over the entire range of optical thicknesses.

It can also be seen in Fig. 3 that, with increasing dispersion ($\Lambda > 0.1$), the $T'_\lambda = f(k_\lambda l)$ curve calculated by formula (24) for $\delta_f = 0.5$, $m = 2$, $\mu = 1$ deviates increasingly from the Buger exponential curve $\exp(-k_\lambda l)$. The curves of spatial irradiance $E_{\lambda,0}$ in Fig. 2 also differ from the exponential curve $\exp(-k_\lambda l)$. When the dispersion is weak ($\Lambda < 0.1$), the Buger–Lambert law can be applied to layer thicknesses up to $k_\lambda l < 3$. In this case the error in the determination of T'_λ neglecting dispersion is up to 5%, but only 3.2% for $k_\lambda l = 1$ and 4.2% for $k_\lambda l = 2$. With increasing dispersion and layer thickness, the error becomes much larger, which is explained by the increasing fraction of multiply dispersed radiation. Thus, with a strong dispersion $\Lambda = 0.9$, the error is less than 20% for $k_\lambda l = 0.5$ but already 75.5% for $k_\lambda l = 3$.

According to Tables 2 and 3, the values of $R_{\lambda\infty}(\mu; 2\pi)$ based on formula (26) for various values of Λ and μ as well as various dispersion indicatrices (spherical $\chi_\lambda(\gamma) = 1$ and the simplest forward elongated $\chi_\lambda(\gamma) = 1 + \cos\gamma$) agree within 1-2% with the results of numerical methods.

Thus, the proposed method of using mean characteristics which do not require that the indicatrix $\chi_\lambda(\gamma)$ be known, makes it possible to determine the radiation inside capillary-porous colloidal or other radiation dispersing materials irradiated by a parallel flux at an incidence angle Θ. In terms of accuracy, this method matches the Ambartsumyan, Hottel, and Tien–Churchill numerical ones.

NOTATION

Λ	is the dispersion factor: life expectance of a quantum;
k_λ	is the absorptivity, m^{-1};
σ_λ	is the dispersivity, m^{-1};
$\chi_\lambda(\gamma)$	is the dispersion indicatrix;
$\bar{k}_\lambda$	is the mean absorptivity, m^{-1};
f_λ	is the mean forward dispersivity, m^{-1};
s_λ	is the mean backward dispersivity, m^{-1};
R_λ	is the reflectance of plane layer, thickness l;
T_λ	is the transmittance of plane layer, thickness l;
A_λ	is the absorbance of plane layer, thickness l;
$R_{\lambda\infty}$	is the reflectance of layer with infinite optical thickness;
Θ	is the incidence angle of radiation flux ($\mu = \cos\Theta$);
E'_λ	is the density of monochromatic radiation impinging on a layer at angle Θ;
q_+, q_-	are the densities of opposing diffuse fluxes inside a layer at depth x;
q'_+, q'_-	are the densities of opposing parallel fluxes inside a layer at depth x;
$E_{\lambda,0} = q_+ + q_-$	is the spatial irradiance due to dispersed flux;
$E'_{\lambda,0'} = q_+ + q_- + q'_+$	is the total spatial irradiance;
$\bar{\varepsilon}_{\lambda,e} = \bar{k}_\lambda + s_\lambda$.	

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